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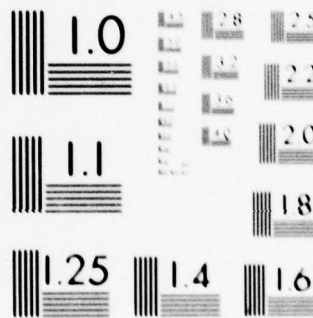
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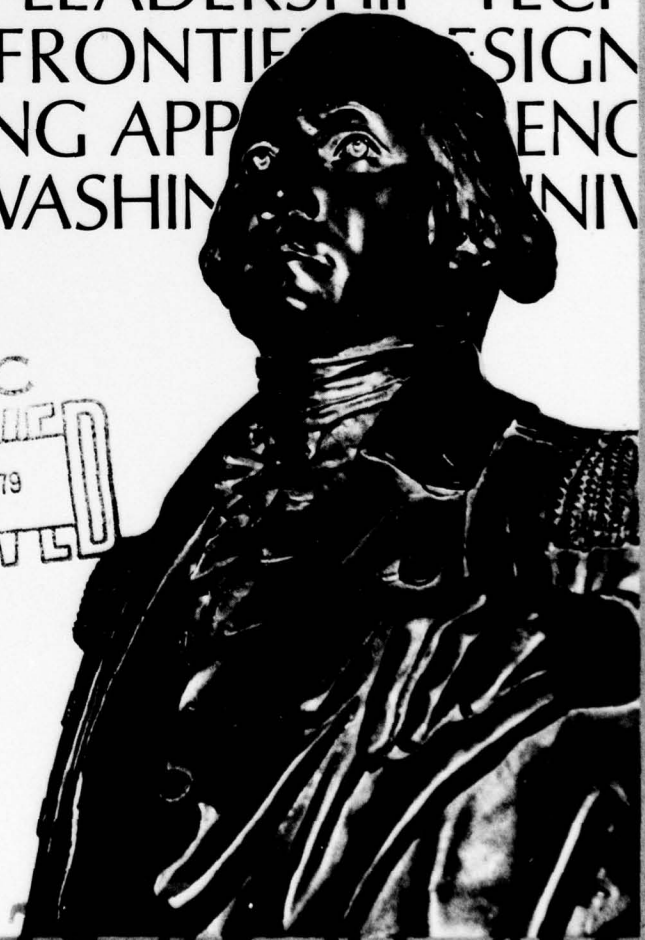
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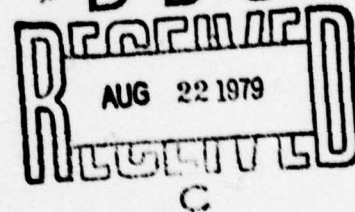
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A SEPARABLE PROGRAMMING APPROACH TO  
THE LINEAR COMPLEMENTARITY PROBLEM

by

Jonathan F. Bard  
James E. Falk

Serial T-403  
22 June 1979



The George Washington University  
School of Engineering and Applied Science  
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The linear complementarity problem (LCP) is reformulated as a nonconvex, separable program and solved with a general branch and bound algorithm. Unlike the principal alternatives, the approach offered here works for all linear complementarity problems regardless of their underlying matrix structure. In the reformulated version, the optimal value is known at the outset so a convergence check can be made at each iteration of the algorithm. This greatly increases its performance; in fact, a number of cases are given where immediate convergence can be expected.

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1. Introduction

Complementarity plays an important role in both general equilibrium theory [1] and mathematical programming. We will be concerned with the linear complementarity problem (LCP) of finding an  $x$  in  $R^n$  such that

$$Mx + v \geq 0, \quad x \geq 0, \quad \langle x, (Mx + v) \rangle = 0 \quad (\text{LCP})$$

where  $M$  is a given  $n \times n$  matrix and  $v$  is a given vector in  $R^n$ . Applications of this problem can be found in such areas as economics, engineering, and game theory (see, for example, [2], [7]). A number of algorithms [4], [7], [9] have been specifically designed to take advantage of the special structure it offers. In each case, however, their applicability is limited by the requirement that  $M$  satisfy certain conditions. In this paper, we offer a solution to LCP that is independent of the structure of  $M$ . Our approach is based on Mangasarian's [8] observation that the linear complementarity problem is equivalent to minimizing a piecewise linear concave function of a polyhedral set contained in the nonnegative orthant; i.e.,



$$\min_{x \in S} \sum_{i=1}^n \{ \min(0, M_i x - x_i + v_i) + x_i \} \quad (1)$$

where  $S = \{x : Mx + v \geq 0, x \geq 0\}$  and  $M_i$  is the  $i$ th row of  $M$ .

In the manner described by Bard and Falk [3], a general branch and bound algorithm is used to solve a separable representation of (1). In fact, if  $Mx + v$  in LCP is replaced by  $g(x)$ , where  $g$  is an implicitly separable function which maps  $R^n$  into itself, the same methodology can be used to solve the more general problem that results.

In the next section, a brief discussion of the branch and bound algorithm is given. Following this, the characteristics of a linear program (LP) equivalent to LCP are presented and a comparison is made between this problem and the series of subproblems set up by the algorithm under the branch and bound philosophy. Special attention is paid to the case where the algorithm can be expected to produce a solution to the linear complementarity problem on its first iteration. Finally, two examples are presented and the results contrasted with alternative solution techniques.

## 2. The Branch and Bound Algorithm

The algorithm that we will use for the computations was proposed by Falk [5] and coded by Grotte [6]. As applied to nonconvex problems with linear constraints, it provides approximate solutions by replacing each of the original functions with their piecewise linear convex envelopes. The branch and bound procedure solves this lower bounding problem first to get estimates on the optimal value of the approximating problem, and to set up new problems, if the estimates do not yield a global solution. When all the original functions are piecewise linear, as they are in (1), the solution will be exact, rather than approximate.

Branch and bound algorithms designed to solve mathematical programs generally produce sequences of upper and lower bounds that converge at the



optimal value. This is, indeed, the case with MOGG (the computer code); however, if a solution to LCP exists, the value of the objective function in (1) at the solution will be zero. Knowing this fact greatly improves the efficiency of the algorithm by permitting an independent check for convergence to be made at each iteration.

### 3. An Equivalent Linear Program

Mangasarian [8] has shown that for any real  $n \times n$  matrix  $M$ , if the solution to LCP exists, it can be obtained by solving the linear program,

$$\min \{cx: x \in S\} \quad (LP)$$

where  $c$  is some suitable vector in  $R^n$ . The following set of conditions (see [8] Theorem 1), given here for completeness, characterizes a suitable  $c$  vector.

$$c = r + M^T s, \quad (r, s) \geq 0 \quad (2.1)$$

$$MZ_1 = Z_2 + vd^T \quad (2.2)$$

$$\langle M, (Y_1 - sd^T) \rangle + \langle Y_2 - rd^T \rangle = 0 \quad (2.3)$$

$$\langle r, Z_1 \rangle + \langle s, Z_2 \rangle - \langle v, (Y_1 - sd^T) \rangle = p^T \quad (2.4)$$

$$\text{diag } p = \text{diag } (Y_1 + Y_2) > 0 \quad (2.5)$$

$$Z_1, Z_2 \in Z, \quad Y_1, Y_2, d, p \geq 0 \quad (2.6)$$

where  $r, s, d, p$  are all in  $R^n$ , and  $Z_1, Z_2, Y_1, Y_2$  are all in  $R^{n \times n}$ ;  $Z$  is the set of all real square matrices with nonpositive off-diagonal elements.

Because of the presence of two bilinear conditions (2.3) and (2.4), it is not easy in general to determine a  $c$  vector for an arbitrary  $M$ . However, for a number of special cases including those when  $M$  is a

Z-matrix, or when  $M$  is strictly or irreducibly diagonally dominant [10] a suitable  $c$  can be obtained through a series of intermediate calculations and the linear complementarity problem can be solved as an ordinary linear program.

Unfortunately, even for these special cases, it is rarely a straightforward matter of identifying the matrices, vectors, and side conditions that are needed to calculate a suitable  $c$  vector. When the dimensions of the problem are greater than three, the work required to determine which linear program to solve begins to rival the work required to obtain a solution to LCP. Those cases where  $c$  can be easily determined are discussed in Section 5.

#### 4. The Relationship Between LP and MOGG

In addressing LCP, MOGG sets up and solves a series of linear programs that closely resemble LP. The constraint region of each subproblem is identical to that of LP, but the cost coefficients vary from iteration to iteration. Eventually, MOGG selects a "correct" set of coefficients and produces a solution. The coefficients are correct only in the sense that the supporting hyperplane (objective function) at the solution of the associated linear program passes through the origin. They are not necessarily equal to the value of a  $c$  in LP as determined by conditions (2.1) - (2.6). There is no guarantee that the objective function in LP evaluated at the solution will be equal to zero. To see this, let us introduce a set of auxiliary variables  $w_i$  ( $i = 1, \dots, n$ ) for the purpose of transforming (1) into a separable programming problem; that is:

$$\begin{array}{ll} \min & \sum_{i=1}^n \{ \min(0, w_i) + x_i \} \\ \text{subject to} & \\ x \in S & \\ w \in W & \end{array} \quad (3)$$

subject to

$$w_i - M_i x + x_i = v_i \quad i = 1, \dots, n$$

where  $W$  is an arbitrarily large hyperrectangle in  $R^n$ .

The iterative procedure used by MOGG to solve (3) was described in Section 2. The equivalent series of linear programs addressed in this procedure can be given in terms of the original variables and a parameter  $\alpha$  in  $R^n$  as follows:

$$\begin{aligned} \min_{\substack{x \in S \\ w \in W}} & \sum_{i=1}^n \{ \alpha_i w_i + x_i \} \end{aligned} \quad (4.1)$$

subject to

$$w_i - M_i x + x_i = v_i \quad i = 1, \dots, n \quad (4.2)$$

where  $\alpha_i$  assumes one of the following three values: 0, 1/2, 1, depending upon which stage the algorithm is in. At the first stage,  $\alpha_i = 1/2$  ( $i = 1, 2, \dots, n$ ); this represents the convex underestimating problem. Although there are  $3^n$  possible combinations of the  $\alpha_i$ 's, some of the associated linear programs turn out to be redundant and are not addressed by MOGG. It is possible to verify through enumeration that  $2^{n+1} - 1$  is the maximum number of subproblems that might have to be solved.

Each auxiliary variable  $w_i$  in problem (4) can be eliminated by substituting its equivalent, as determined from (4.2), into (4.1), and noting that  $W$  is arbitrarily large. Lemma 2 in [8] assures that the solution of LCP occurs at a vertex of  $S$ . The resulting problem is

$$\begin{aligned} \min_{x \in S} & \sum_{i=1}^n \{ x_i + \alpha_i (M_i x - x_i + v_i) \} \\ \Leftrightarrow & \min_{x \in S} \sum_{i=1}^n \{ (1-\alpha_i)x_i + \alpha_i M_i x \}, \end{aligned} \quad (5)$$

which has the same constraint region as LP; hence, any solution to LP



will be both feasible and optimal to (5). This leads to the following lemma which characterizes immediate solutions to LCP.

LEMMA 1. Let the linear complementarity problem have a solution, and let the objective function  $c$  of the associated linear program satisfy conditions (2.1) - (2.6). Now, if, for some  $\gamma > 0$ ,

$$c_j = \gamma \left( 1 + \sum_{i=1}^n M_{ij} \right), \quad j = 1, 2, \dots, n \quad (6)$$

then Falk's algorithm will produce a solution to either problem on its first iteration.

This can be seen by letting  $\alpha_i = 1/2$  ( $i = 1, \dots, n$ ) and equating the cost coefficients of LP and (5). The applicability of this result is more general than it would first appear because  $c$  will usually assume a range of values.

In fact, (6) is only a sufficient condition for the algorithm to produce a solution on its first iteration. A necessary and sufficient condition would be that the vector  $\gamma \left( 1 + \sum_{i=1}^n M_{ij} \right)$ ,  $i = 1, 2, \dots, n$  lie in the cone formed by the gradients of the binding constraints of the associated linear program. This condition, of course, is untestable in that the calculation of  $c$  offers no hint as to which constraints will be binding at the solution.

## 5. A Verifiable Case

In general, even if a suitable  $c$  is known, the only way to determine if MOGG will produce a solution to LCP on its first iteration is by evaluating (6). In this section we examine the special case where  $c$  assumes a unit structure and show that in this instance the solution is immediate. A statement of this result is contained in the following theorem.

THEOREM 1. Let  $x^*$  solve LCP. If  $M$  is such that  $c = \beta e$



satisfies conditions (2.1) - (2.6) for some  $\beta > 0$  and  $e = (1, 1, \dots, 1)$  then MOGG will produce a solution to LCP on its first iteration.

PROOF: Let  $(x^*, u_1^*, u_2^*)$  in  $R^n \times R^n \times R^n$  solve LP, where  $u_1^*$  and  $u_2^*$  are the Kuhn-Tucker multipliers associated with the inequality constraints  $Mx^* + v \geq 0$  and  $x^* \geq 0$ , respectively. It will be shown that there exists a corresponding point  $(x^*, \bar{u}_1, \bar{u}_2)$  in  $R^n \times R^n \times R^n$  that satisfies the Kuhn-Tucker conditions for (5) with  $\alpha_i = 1/2$  ( $i = 1, \dots, n$ ) and is thus the solution to the first subproblem set up by MOGG.

Letting  $c = \beta e$ , the first order necessary conditions for LP that require that

$$\beta \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = u_{11}^* \begin{pmatrix} M_{11} \\ \vdots \\ M_{1n} \end{pmatrix} + \dots + u_{1n}^* \begin{pmatrix} M_{n1} \\ \vdots \\ M_{nn} \end{pmatrix} + u_{21}^* \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \dots + u_{2n}^* \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix} \quad (7)$$

if  $(x^*, u_1^*, u_2^*)$  is to be a solution. Similarly for (5) with  $\alpha_i = 1/2$

$$\frac{1}{2} \begin{pmatrix} 1 + \sum_{i=1}^n M_{ij} \\ \vdots \\ 1 + \sum_{i=1}^n M_{in} \end{pmatrix} = \bar{u}_{11} \begin{pmatrix} M_{11} \\ \vdots \\ M_{1n} \end{pmatrix} + \dots + \bar{u}_{1n} \begin{pmatrix} M_{n1} \\ \vdots \\ M_{nn} \end{pmatrix} + \bar{u}_{12} \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \dots + \bar{u}_{2n} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}. \quad (8)$$

Multiplying (8) by  $2\beta$  and rearranging we get

The following example [8] illustrates the equivalence stated in Theorem 1 while concurrently demonstrating the impracticality of casting the linear complementarity problem as a linear program when  $c$  is not explicitly given. The example is based on the following theorem.

THEOREM 2 (Mangasarian [3]). If  $S \neq \emptyset$  and there exist  $r, s$  in  $\mathbb{R}^n$ ,  $Z_1, Z_2$  in  $\mathbb{R}^{n \times n}$  such that

$$MZ_1 = Z_2 + vd^T$$

$$\langle r, Z_1 \rangle + \langle s, Z_2 \rangle \geq 0$$

$$\langle r, (Z_1 + D) \rangle + \langle s, (Z_2 + D) \rangle > 0, \quad D = \text{diag } d$$

$$Z_1, Z_2 \text{ in } Z; d, r, s \geq 0$$

then LCP has a solution which can be obtained by solving LP with  $c = r + M^T s$ .

EXAMPLE 1.

$$M = \begin{pmatrix} 0 & 3 & 4 \\ -1 & -1 & 0 \\ 2 & -1 & -3 \end{pmatrix} \quad \text{and} \quad v = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}.$$

This example satisfies the conditions of Theorem 2 with  $d = s = e$ ,  $r = 0$ ,

$$Z_1 = \begin{pmatrix} -0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \text{and} \quad Z_2 = \begin{pmatrix} 2 & -1 & -2 \\ -1 & 0 & 0 \\ -1 & 0 & 2 \end{pmatrix}.$$

Now  $c = r + M^T s = 0 + M^T e = e$ ; hence, by Theorem 1 with  $\beta = 1$  (or Lemma 1 with  $\gamma = 1/2$ ) we have the equivalence of (5) and LP. As expected, the first iteration of the algorithm produced the equilibrium point  $x^* = (2/5, 2/5, 1/5)$ . It is interesting to note that this problem cannot be solved by either Lemke's method or the principal pivoting procedure [4].

The next example [8] illustrates the case where a solution to the

linear complementarity problem is not obtained on the first iteration of MOGG.

EXAMPLE 2.

$$M = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix} \text{ and } v = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

This example also satisfies the conditions of Theorem 2 with

$$r^T = (0,1), s^T = (1,0), d^T = (0,2), z_1 = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 2 \end{pmatrix}, z_2 = \begin{pmatrix} -\frac{1}{2} & 0 \\ -1 & 0 \end{pmatrix},$$

and  $c = (-1,2)$ . The associated linear program has the (unique) solution  $x_1 = 1, x_2 = 0$  which expectedly solves the linear complementarity problem. When MOGG was used to solve this problem, the solution was found after three stages of branching had taken place. The branch and bound tree is depicted in Figure 1, and, as can be seen, six of the seven potential subproblems had to be examined before convergence could be established. (The numbers adjacent to the nodes represent the upper and lower bounds for the associated subproblems.)

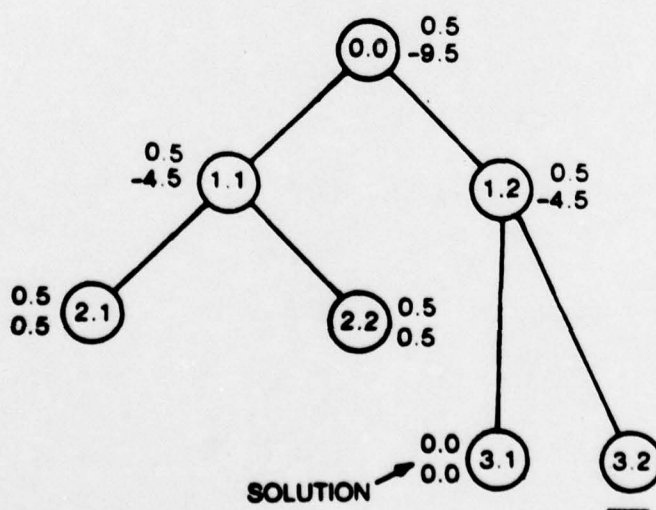


Figure 1 Branch and Bound Tree for LCP Example 2



When Lemke's method was tried on this problem, an unbounded (infeasible) ray was generated by letting  $x_2 \rightarrow \infty$ , thus precluding a solution. The principal pivoting algorithm also ran into trouble by cycling rather than converging to solution.

From these two examples, we see that MOGG offers a clear advantage in solving the linear complementarity problem over the principal alternatives. Because MOGG does not insist upon a special matrix structure, it will solve all such problems without first having to check the properties of  $M$  or evaluate an often unwieldy set of nonlinear conditions.

In general, the branch and bound approach would appear to offer the dual advantage of being universally applicable, and computationally superior. This results directly from the guaranteed upper bound, and the simple three-segment form of the objective function.



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A VAULT FOR THE FUTURE  
IN THE YEAR 2036

THE STORY OF ENGINEERING IN THIS YEAR OF THE PLACING OF THE VAULT AND  
ENGINEERING HOPES FOR THE TOMORROWS AS WRITTEN IN THE RECORDS OF THE  
FOLLOWING GOVERNMENTAL AND PROFESSIONAL ENGINEERING ORGANIZATIONS AND  
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